2. The rotational energy of beam electrons increases with increasing current, and may reach an appreciable fraction of the total energy. The rotational energy decreases with an increase in the magnetic field, and approaches zero in the limit of an infinitely large field.

3. State diagrams relating the energy characteristics of the beam, the current, and the geometrical parameters enable one to distinguish three regions differing in the initial state. The transition from one region to another for an adiabatically slow variation of the magnetic field enables one to produce different transformations of the energy of the system, ensuring an adiabatic acceleration and deceleration, and also an appreciable compression of the beam within the framework of the model considered.

The authors thank A. V. Zharinov for posing the problem and for a discussion of the paper.

LITERATURE CITED

1. R. Davidson, Theory of Nonneutral Plasmas, Benjamin, New York (1974).
2. P. Sprangle, A. T. Drobot, and W. M. Manheimer, "Collective ion acceleration in converging guide," Phys. Rev. Lett., 36, 1180 (1976).
3. V. P. Fotin, V. V. Glebov, V. A. Malafaev, D. N. Novichkov, V. I. Perevodchikov, and A. V. Zharinov, "Simulation of phenomena associated with transport of decompensation and quasi-neutral electron beams for industrial scale transmission of electric power," in: Fourth Intern. Topical Conf. on High-Power Electron and Ion Beams, Palaiseau (1981).
4. L. P. Smith and P. L. Hartman, "The formation and maintenance of electron and ion beams," J. Appl. Phys., 11, 220 (1940).
5. L. É. Bakhrakh, "Limiting current of an electron beam under the assumption of a constant space charge density," Radiotekh. Elektron., 10, No. 6 (1965).
6. A. V. Zharinov, D. N. Novichkov, and A. S. Chikhachev, "Equations of state of an electron beam," Radiotekh., Elektron., 23, No. 8 (1978).
7. B. N. Breizman and D. D. Ryutov, "Powerful relativistic electron beams in a plasma and in a vacuum (theory)," Nucl. Fusion, 14, 873 (1974).
8. L. Brillouin, "A theorem of Larmor and its importance for electrons in magnetic fields," Phys. Rev., 67, 260 (1945).

NUMERICAL CALCULATIONS OF STATIONARY STATES OF MAGNETIC SELF-INSULATION OF VACUUM LINES

G. G. Golovin, A. V. Gordeev
B. D. Korolev, V. P. Smirnov,
and A. S. Chernenko

UDC 533.916 : 517.949.8

Magnetic self-insulation of vacuum gaps permits attaining electric fields of $> 10^6$ V/cm due to screening of the negative electrode by a layer of magnetized electrons [1]. As a result, it is possible to transmit energy fluxes along vacuum lines and to concentrate them to densities $\geqslant 10^{12}$ W/cm^2, which finds application, in particular, in large-scale systems, for example, Angara-5 [2]. In spite of the broad practical application of self-insulation, there is as yet no complete theory of the equilibrium of electron layers. The best developed models are the hydrodynamic Brillouin model and the kinetic model with one type of trajectory. The hydrodynamic model, which does not tale into account the pressure in the electron layer (Brillouin flow), describes well cylindrical lines. The more realistic kinetic model, which takes into account one type of electron trajectory, predicts the existence of equilibrium configurations only for flat and cylindrical lines and, in addition, in the latter, the external electrode must be negative [3]. The important case of converging conical lines, which is important for concentrating energy flux, is described only approximately by the hydrodynamic model. In the self-consistent kinetic as well as in the single-frequency approximations, there are no solutions, which is a result of the dependence of the azimuthal magnetic field on the distance to the apex of the cone [4]. Great diffi-

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 31-37, September-October, 1983. Original article submitted August 9, 1982.

 0021-8944/83/2405-0632$07.50

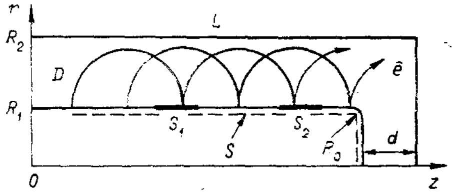

Fig. 1

culties likewise arise in attempts to describe analytically the coupling between the line and the diode at the end of the line. This problem is a particular case of the problem of inhomogeneous lines, for which approximate computational methods are only now being formulated. As an example, we point to the adiabatic model of the high-current diode [5]. The present situation urgently requires the development of numerical methods for calculating lines with different configurations. At the present time, calculations based on the stabilization method using macroparticles are used for such problems. However, direct application of the stabilization method to calculation of a line leads to large fluctuations, which greatly reduce accuracy, overload computer memory, and increase computational time. A new stationary numerical algorithm described in this paper eliminates the fluctuations and works practically with any electrode. Results of numerical calculations of lines with magnetic insulation are presented here for two configurations (cylindrical and conical) and are compared with experiment.

The most general geometry of a vacuum coaxial line is shown in Fig. 1. The inner cylinder with radius R_1 is the cathode and the outer cylinder with radius R_2 is the anode. The sharp edge of the cathode at the end of the coaxial line is rounded and the rounding radius R_0 is approximately 1% of R_1. We shall denote the magnitude of the accelerating gap at the outlet of the coaxial line by d. For large values of the potential difference U applied to the electrodes, explosive electron emission arises on part of the cathode S (in Fig. 1, it is shown by a dashed line) [6]. The explosive emission results in a vanishing normal component of the electric field intensity E_n on the surface S for steady-state electron motion: $E_n = 0$. This condition serves to determine the emission current density j_e on S.

The stationary motion of the relativistic electron plasma in the vacuum coaxial line can be described with the help of the system of Maxwell's equations and Vlasov's equation:

$$\begin{gathered}\operatorname{div}\mathbf{E} = 4\pi\rho = -4\pi e\int f d^3p,\\ \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}j = -\frac{4\pi}{c}e\int \mathbf{v} f d^3p,\\ \operatorname{div}\mathbf{H} = 0, \quad \operatorname{rot}\mathbf{E} = 0,\\ (\mathbf{v}\nabla)f - e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\mathbf{H}]\right)\frac{\partial f}{\partial p} = 0\end{gathered} \tag{1}$$

with fixed boundary conditions. In the system (1), E and H are the intensities of the electric and magnetic fields; ρ and j are the charge and current densities; f is the electron distribution function; −e and m are the charge and mass of the electron; c is the velocity of light; the momentum p and the velocity v of the electrons are related by the relation

$$v = \frac{p}{m}\left(1 + \frac{p^2}{m^2c^2}\right)^{-1/2}.$$

In view of the difficulty of integrating Vlasov's kinetic equation, in practice, the equations of motion of separate particles are usually integrated:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad \frac{dp}{dt} = -e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\mathbf{H}]\right),$$

which are characteristics of the kinetic equation. In this case, the system of equations (1) in cylindrical coordinates r, θ, and z has the form presented in [7].

1. Stationary Maxwell's equations

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\varphi}{\partial r}=-4\pi\rho; \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)=\frac{4\pi}{c}j_z, \quad -\frac{\partial H_\theta}{\partial z}=\frac{4\pi}{c}j_r \tag{3}$$

with boundary conditions for the potential φ and the component of the magnetic field intensity H_θ

$$\varphi|_K=0, \quad \varphi|_A=U, \quad \left.\frac{\partial\varphi}{\partial r}\right|_{r=0}=0, \tag{4}$$

$$\varphi\Big|_{\substack{z=0\\R_1<r<R_2}}=U\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}, \quad H_\theta|_{r=0}=0.$$

2. Equation of motion of the electrons

$$\frac{dv_z}{dt}=-\frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left\{E_z+\frac{1}{c}v_rH_\theta-\frac{1}{c^2}v_z(v_zE_z+v_rE_r)\right\}, \tag{5}$$

$$\frac{\partial v_r}{dt}=-\frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left\{E_r-\frac{1}{c}v_zH_\theta-\frac{1}{c^2}v_r(v_zE_z+v_rE_r)\right\},$$

$$\frac{dr}{dt}=v_r, \ \frac{dz}{dt}=v_z, \quad E_r=-\frac{\partial\varphi}{\partial r}, \quad E_z=-\frac{\partial\varphi}{\partial z}$$

with initial conditions

$$r(s,0)=r^0(s), \ z(s,0)=z^0(s),$$

$$v_r(s,0)=v^0(s)\cos(e_r, n(s)), \ v_z(s,0)=v^0(s)\cos(e_z, n(s)),$$

where v°, r°, z° are constants that are given in the formulation of a specific problem; n(s) is the direction of the normal in the D region to the cathode at the point of the cathode; s, e_r, and e_z are the unit vectors of the coordinate axes; $v^2 = v_r^2 + v_z^2$; and s is the coordinate along the surface S.

3. The relations for determining the functions ρ and j at any point M in the region D are as follows:

$$\rho(M)=\frac{eN(V_M)}{V_M}, \quad j(M)=\frac{e\sum_{k=1}^{N(V_M)}v_k}{V_M}=\rho(M)v(M), \tag{6}$$

where V_M is a volume of fixed size and shape surrounding the point M; $N(V_M)$ is the number of electrons in this volume; and v_k is the velocity of the electrons.

The number of equations of motion (5) in the system written out is very large and coincides with the number of electrons located in the gap between the electrodes. It is impossible to integrate such a system in practice. However, if large groups of electrons with close trajectories of motion are combined into macroparticles, then the number of equations of motion (5) in the system (2)-(6) is considerably reduced, which makes the problem of its numerical integration realistic.

We note that in using the method of macroparticles, the system (2)-(6) does not change. Only the method of counting the number of electrons $N(V_M)$ in the volume V_M in expression (6) changes; this method is related with one or another method for "smearing" the macroparticles [8].

To describe the motion of electrons in a vacuum coaxial line with the help of the system of equations (2)-(6), it is necessary to know their initial velocities, the magnitude of the electron flux from the cathode in the region D (the emission current density j_e), and the region of emission S. We shall assume that in the physical problem formulated above, the initial velocity v° of the electrons is identical for all points on the cathode, is oriented perpendicular to the cathode surface, and is of the order of $v^0 \simeq 10^7$ cm/sec [6].

To determine the emission current density j_e, the condition on the normal component of the electric field intensity at the cathode is used:

$$E_n|_S = 0, \tag{7}$$

which is a result of the infinite emissivity of the cathode. In this paper, we shall assume that the emission zone S consists of the part of the cathode surface where the normal component of the electric field intensity in the absence of volume charge exceeds a critical magnitude $E_*(E_* \simeq 150-200$ kV/cm), for which explosive emission appears [6]. To determine this emission zone, it is necessary to find the solution of Laplace's equation in the absence of emission with boundary conditions [4].

Some sections of the emission zone S can be screened by the electron flux arriving from other parts of the zone (see Fig. 1, sections S_1 and S_2). For this case, it was necessary to use a more general condition

$$E_n|_S \geqslant 0, \tag{8}$$

which permits screening of any parts of the cathode.

We note that the system (2)-(7) permits determining the self-consistent electromagnetic fields E and H, the motion of charged particles in them, and the emission current density j_e at the cathode.

We shall briefly list the basic details of the numerical algorithm used in this work (the algorithms are described in greater detail in [7, 9]).

1. Poisson's (2) or Laplace's equation with zero boundary conditions or with conditions (4) is solved by the difference method. For this, a considerably nonuniform spatial grid is induced in region D (see Fig. 1): The steps near the cathode (along the normal to its surface), near the cathode corner (along its surface), and near the edge of the emission zone (along the cathode surface) are 10^3-10^4 times smaller than the maximum step in the grid in region D. The algebraic system of equations obtained when the Laplacian operator is replaced by its difference analog is solved by the method of matrix factorization.

2. The quantities E_r and E_z are determined from the formulas for numerical differentiation of two successive equations in group (5). To determine the quantity E_n at the cathode, the values of the potential on three nodes of the grid near the boundaries, one of which is a boundary node, are used.

3. To determine H_θ in the accelerating gap, the first of equations (3) is used and the second equation in (3) is used to determine H_θ in the gap between the cylindrical surfaces.

4. The equations of motion (5) are integrated with the help of Euler's method with one correction [7].

5. To calculate the quantities $N(V_M)$ in Eqs. (6), the point-smearing method is used [8].

6. The method of test charges, proposed in [7], is used to determine the quantities j_e from conditions (7) or (8).

7. The system of equations (2)-(8) is integrated by the method of iteration [7, 9]. The distinguishing feature of the iterative method used is satisfaction of the condition (7) at each step, which guarantees rapid convergence of the method.

Five improperly posed partial problems were discovered in this work: the problems for determining the current density and the charge density inside the region; the problem of determining j_e at the cathode from condition (7) [10]; the stationary motion relative to small perturbances of the electric and magnetic fields is unstable [11]. Each of the difficulties indicated requires its own method of regulation. Condition (7) is an effective regulator of the basic electrical instability. In the problem being examined, there is a power-law singularity of the right side of Poisson's equation (2). The use of a nonuniform grid considerably increases the accuracy of the solutions near the singularity and eliminates their "agitation" [10-12].

For short segments of the lines, the magnitude of the total current turned out to be proportional to the length of the emitting surface, which agrees with the results in [13].

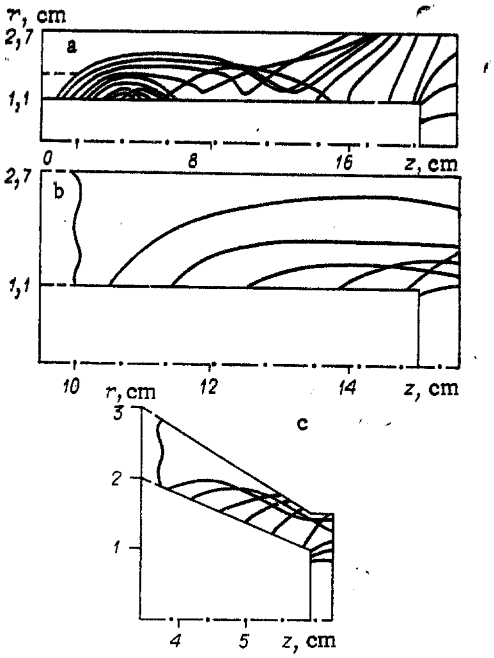

Fig. 2

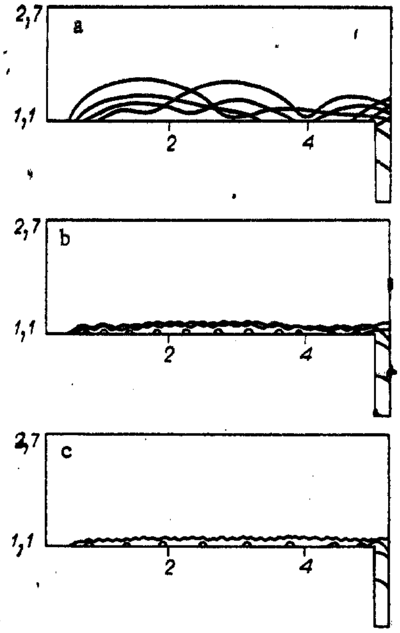

Fig. 3

When the length of the line L increases, due to screening of the emitting surface, this proportionality breaks down and the total current in the line I_l no longer depends in the length. This current coincides with the self-insulation current of the line, at which the leakage currents on the outer electrode vanish. For a line with the radii of the outer R_2 and inner R_1 electrodes forming the ratio R_2/R_1 = 2.7 cm/1.1 cm and with negative voltage on the cathode U = 400 kV, the current I_l remained practically unchanged with increasing emission zone, beginning with L > 10 cm.

Figure 2 shows the trajectories of electrons in the case of the steady-state insulation regime. When the voltage on the line U = 410 kV and while the accelerating gap d = 2 cm (Fig. 2a), the computed total current of the line equals 17.8 kA. For a voltage of U = 390 kV on the line and the gap d = 0.6 cm (Fig. 2b), the current equals 18.8 kA. As is evident from Fig. 2a, the motion of particles far from the end face of the line occurs in the form of an electron layer, pressed to the inner electrode. The height of this layer exceeds the height obtained from the hydrodynamic approximation [1], which could be related with the initial spread in the velocities of the particles. Thus, according to the hydrodynamic theory, for the current and voltage indicated, the height of the layer equals 0.7 cm (the dashed straight line in Fig. 2a), while computed height is 1.0 cm.

Figure 3 shows the trajectories of electrons as a function of the magnitude of the applied voltage for constant accelerating gap d = 0.2 cm (R_2/R_1 = 2.7 cm/1.1 cm). In Fig. 3a-c, the voltage and current of the line equal, respectively, 100 kV and 10 kA, 200 kV and 27 kA, and 400 kV and 76 kA. As is evident from Fig. 3, an increase in voltage leads to narrowing of the effective emission zone on the surface of the negative electrode, participating in the creation of the current in the line, and to a decrease in the height of the layer.

Experiments were performed on the MS accelerator (U = 350 kV, I = 35 kA, τ = 40 nsec) [1, 14].

The computed magnitudes of the total current in the line I_l and the current I_a, reaching the end anode, are compared with the experimental values as a function of the accelerating gap d in Fig. 4. The dark points are the experimental values and the open circles correspond to the maximum values of I_l and I_a. It follows from the experiment and from the computational results that for $d \leqslant 0.6$ cm, the current in the line is completely switched at the end anode (see also Fig. 2b). As the gap increases (d > 0.6 cm), part of the electron flux reaches the outer electrode of the line. For d > 1 cm, the experimetnal and computed values of the total current in the line approached their limiting value, which coincides to within 10% with the theoretical minimum current I_{min}, required to establish the insulation state [1]. In this case, practically the entire current (I_l = 17.5 kA) is short-circuited through the outer electrode at the end of the line (14 kA) in the region $13 \leqslant z \leqslant 17$ cm.

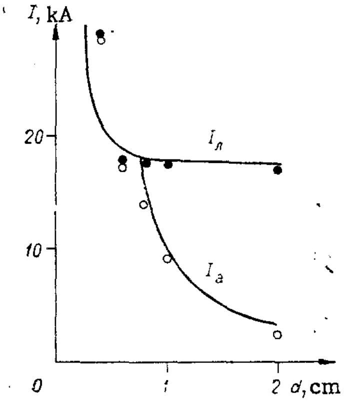

Fig. 4

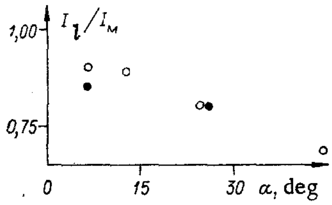

Fig. 6

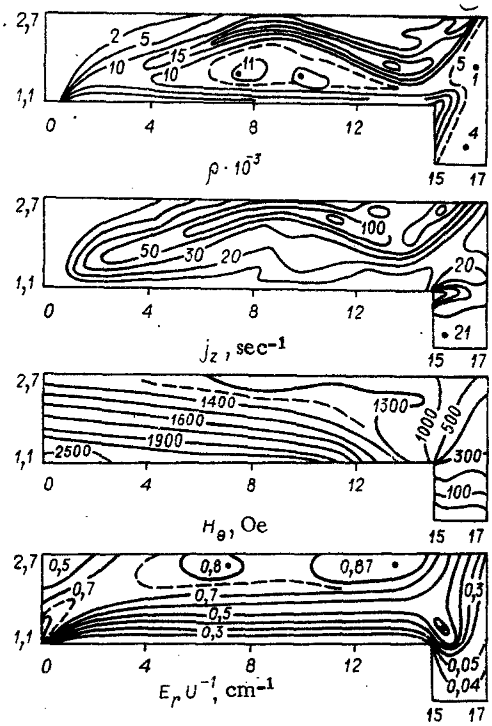

Fig. 5

It should be noted that for small accelerating gaps ($d \leqslant 0.4$ cm), the experimental and computed dependences of I_l and I_a disagreed. This is apparently related with the decrease in the effective gap d as a result of the motion of the near-electrode layers of the plasma. The disagreement was eliminated by assuming that the velocity of the motion of the plasma equals $(1.5–2)\cdot10^6$ cm/sec. A limiting measured current equal to 17.5 kA is attained in calculations with an emission length $S \geqslant 10$ cm. For L = 17 cm and d = 2 cm, a current equal to 1.8 kA flows from the end cathode ($0 \leqslant r \leqslant 1.09$ cm) to the anode; a current of 0.66 kA flows from the rounded part of the cathode (R_0 = 0.01 cm); and, a current of 15.8 kA flows from the cylindrical surface. In addition, a current of 8.2 kA flows from the surface adjacent to the end cathode ($10 \leqslant z \leqslant 15$ cm) and a current of 7.6 kA flows from the remaining surface ($z \leqslant 10$ cm). We note that as the calculations show, beginning with some value R_0, the total current is nearly independent of the decrease in the rounding radius of the cathode. It follows from the calculation that the maximum values of the leakage current, as well as of the current density, occur on the region of the surface of the outer electrode ($14 \leqslant z \leqslant 16$ cm) and are localized near the end surface of the cathode in the line (see Fig. 2b). This effect was observed experimentally [15].

Figure 5, which shows the lines indicating the level of the four functions ρ, j_z, H_θ, and E_r/U (the dependences are given in cgs units), illustrate the distribution of charge ρ, axial current density j_z, and the magnetic H_θ and electric E_z fields in the vacuum line.

Together with the coaxial vacuum lines, calculations were performed for conical uniform lines with the cone angle of the cathode varying from 5 to 37.5° and the angular gap between the electrodes varying from 3 to 12.5°. The length of the lines varied from 3 to 25 cm, while the ratios of the input and output radii of the electrodes constituted R_{20}/R_{10} = 5.0 cm/3.3 cm and R_2/R_1 = 1.5 cm/1.0 cm, respectively. Figure 2c shows the results of the numerical calculation of electron trajectories, corresponding to the stationary flow, with a voltage of 160 kV on the line for an accelerating gap of 0.3 cm. As in the case of cylindrical lines, a gap d = 0.2 cm existed for which the entire current was switched at the end anode; in addition, in the experiments, this switching was observed with somewhat larger gaps ($d \simeq 0.3$ cm) than suggested by calculations.

Figure 6 shows the dependence of the limiting current in the conical line with constant voltage as a function of the cone angle (open circles show the experimental values and filled circles show the computed values). It is evident that as the cone angle of the line increases, the values of the limiting current decrease compared with the minimum values.

The measured limiting current, as the computational and experimental results show, depends on the cone angle of the line. This cannot be explained in an analytic equilibrium model within the framework of the Brillouin approximation.

In this work, an effective numerical method for calculating vacuum high-voltage systems in the absence of an external magnetic field along the direction of propagation of the current has been created for the first time in computational practice in this country. It is well known that the presence of a high longitudinal magnetic current leads to stabilization of fluctuations and facilitates the calculations [16].

With the help of the procedure described above, calculations of a number of variants of cylindrical and conical vacuum lines were performed. As a result of these calculations, the structure of electron flows in lines was investigated in detail, and the trajectories of charged particles as well as the distribution of the fields and of the current density both in the vacuum region and at the boundaries of the electrodes were obtained. The computed integral characteristics of the beams agree well with the results of physical experiments.

LITERATURE CITED

1. E. I. Baranchikov, A. V. Gordeev, et al., "Magnetic self-insulation of electron beams in vacuum lines," Zh. Eksp. Teor. Fiz., 75, No. 6 (1978).
2. L. I. Rudakov, "Transport of relativistic electron plasma up to a thermonuclear target," Fiz. Plazmy, 4, No. 1 (1978).
3. A. V. Gordeev, "Possibility of breakdown of magnetic self-insulation," Pis'ma Zh. Tekh. Fiz., 3, No. 16 (1977).
4. A. I. Fedosov, "The problem of magnetic self-insulation," in: Abstracts of Reports at the 3rd All-Union Symposium on High-Current Electronics, Izd. Institute of High-Current Electronics, Sib. Otd. Akad. Nauk SSSR, Tomsk (1978).
5. B. N. Breizman and D. D. Ryutov, "Theory of focusing of relativistic electron beam in a diode," Dokl. Akad. Nauk SSSR, 225, No. 6 (1975).
6. S. P. Bugaev, E. A. Litvinov, et al., "Explosive electron emission," Usp. Fiz. Nauk, 115, No. 1 (1975).
7. G. T. Golovin, "Numerical investigation of the stationary motion of a relativistic electron plasma in a cylindrical vacuum diode using the macroparticle method," in: Difference Methods of Mathematical Physics [in Russian], edited by A. A. Samarskii and Yu. P. Popov, Izd. MGU, Moscow (1979).
8. A. V. Zakharov, A. A. Samarskii, and A. G. Sveshnikov, "Application of the method of large particles to calculation of the motion of a charged beam in an electromagnetic field taking into account the space charge of the beam," in: Computational Methods and Programming [in Russian], Izd. MGU, Moscow (1971), No. 16.
9. G. T. Golovin, A. V. Gordeev, I. S. Gushchin, A. V. Zakharov, and Yu. P. Mokin, "Modeling of relativistic electron beams in high-current vacuum diodes," in: Proceedings of 3rd International Conference on Powerful Electron and Ion Beams, Izd. IYa Sib. Otd. Nauk Akad. SSSR, Novosibirsk (1979).
10. Yu. I. Mokin, "Characteristics of improperly posed problems arising in the investigation of electron motion in an electromagnetic field," Zh. Vysshl. Mat. Mat. Fiz., 20, No. 1 (1980).
11. G. T. Golovin, "An iterative method for integrating stationery systems of equations of electrodynamics, related with inclusion of volume charge," in: Computational Methods and Programming [in Russian], Izd. MGU, Moscow (1979), No. 31.
12. G. T. Golovin, "An iterative method for solving diode problems," in: Difference Methods of Mathematical Physics [in Russian], Izd. MGU, Moscow (1980).
13. A. A. Kolomenskii, V. G. Krastelev, and B. N. Yablokov, "Measurement of the self-insulation current of vacuum coaxial lines," Pis'ma Zh. Tekh. Fiz., 3, No. 13 (1977).
14. L. E. Aranchuk, E. I. Baranchikov, A. V. Gordeev, et al., in: Proceedings of International Conference on Powerful Electron and Ion Beams, Izd. IYaF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1979), Vol. 2.
15. L. E. Aranchuk, E. I. Baranchikov, A. V. Gordeev, V. D. Korolev, and V. P. Smirnov, "Effect of ion leakage on magnetic self-insulation of electrons in a vacuum coaxial line," IAÉ-3251/7, Moscow (1980).
16. Yu. A. Berezin, B. N. Breizman, and V. A. Vshivkov, "Numerical modeling of injection of a powerful electron beam into a vacuum chamber with a strong magnetic field," Preprint No. 18, ITPM Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1979).